4.2 Enrichment and Extension

Adding, Subtracting, and Multiplying Polynomials

In Exercises 1–6, use the properties of adding, subtracting, and multiplying polynomials to solve for the variables.

1. When \( ax^2 + bx - 1 \) is added to \( ax^3 + 2bx^2 - 2x - 1 \), the result is \( 3x^3 + 11x^2 + 2x - c \). Find \( a, b, \) and \( c \).

2. When \( 2bx^4 + x^2 - b \) is added to \( 3x^3 - dx^2 + cx + a \), the result is \( 6x^4 + bx^3 - 4x^2 + 2x - 2 \). Find \( a, b, c, \) and \( d \).

3. When \( bx^4 - x^2 + 2x - 4 \) is subtracted from \( 7x^4 + cx^2 + 5x + 6 \), the result is \( 5x^4 + ax^3 - 3x^2 + 3x + d \). Find \( a, b, c, \) and \( d \).

4. When \( 5x^3 - 3ax + 6 \) is subtracted from \( 10x^3 + ax^3 - x + b \), the result is \( cx^3 - 4x^2 + dx - 3 \). Find \( a, b, c, \) and \( d \).

5. The expression \( 35x^3 + 21x^4 + 7x^3 \) is \( ax^3 \) times greater than \( 5x^3 + (3 + b)x + c \). Find \( a, b, \) and \( c \).

6. The expression \( ax^2 - bx + 25 \) is \( 3x - c \) times greater than \( 3x - c \). Find \( a, b, \) and \( c \).

7. Complete the missing values and bottom row of Pascal’s Triangle.

\[
\begin{array}{cccccc}
1 \\
1 & 1 \\
1 & 2 & 1 \\
1 & 3 & 3 & 1 \\
1 & 4 & 6 & 4 & 1 \\
1 & 5 & 10 & 10 & 5 & 1 \\
& 6 & 15 & 20 & 15 & 6 & 1 \\
\end{array}
\]

In Exercises 8–13, use Pascal’s Triangle to expand \((a + b)^n\) with \( n = 6 \), for the \( a-b \) pair given.

8. \( a = x \) and \( b = 1 \)

9. \( a = 2y \) and \( b = -2 \)

10. \( a = 1 \) and \( b = -y \)

11. \( a = x^2 \) and \( b = 0 \)

12. \( a = x^2 \) and \( b = -2 \)

13. \( a = bc \) and \( b = de \)
1. \(ax^2 + bx - 1\) 
   \(\frac{a x^3 + 2b x^2 - 2x - 1}{3x^3 + 11x^2 + 2x - c}\) 
   \(a = 3\) 
   \(b = 4\) 
   \(c = 2\)

   \(b - 2 = 2\) 
   \(b = 4\) 
   \(c = 2\)

2. \(2b x^4 + x^2 - b\) 
   \(\frac{3x^3 - c x^2 + cx + a}{6x^4 + bx^3 - 4x^2 + 2x - 2}\) 
   \(a = 1\) 
   \(b = 3\) 
   \(c = 2\) 
   \(d = 5\)

   \(2b = 6\) 
   \(1 - d = -4\) 
   \(-3 + a = -2\)

   \(b = 3\) 
   \(-d = -5\) 
   \(+3 + 3\)

   \(d = 5\) 
   \(a = 1\)

3. \(7x^4 \downarrow + c x^2 + 5x + b\) 
   \(-b x^4 \downarrow + x^2 - 2x + y\) 
   \(\frac{5x^4 + ax^3 - 3x^2 + 3x + d}{7 - b = 5\) 
   \(c + 1 = -3\) 
   \(-b = -2\) 
   \(-1 - 1\)

   \(b = 2\) 
   \(c = -4\)
8-13 7 spaces - 7th line of Pascals

\[ a = x \]
\[ b = 1 \]

\[ x^6 + 6x^5 + 15x^4 + 20x^3 + 15x^2 + 6x + 1 \]

9)
\[ a = 2x \]
\[ b = -2 \]

\[ \frac{\binom{32}{2}}{64y^6} - \frac{384y^5}{960y^4} - \frac{128y^3}{960y^2} \]

\[ 64y^6 - 384y^5 + 960y^4 - 1280y^3 + 960y^2 - 384y + 64 \]

10)
\[ a = \frac{1}{i} \]
\[ b = -y \]

\[ 1 - 6y + 15y^2 - 20y^3 + 15y^4 - 6y^5 + y^6 \]

11)
\[ a = x^2 \]
\[ b = 0 \]

\[ x^{12} \]

12)
\[ a = x^2 \]
\[ b = -2 \]

\[ \chi^{12} - 12x^{10} + 60x^8 - 160x^6 + 240x^4 - 192x^2 + 64 \]